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Isospin amplitudes and CP violation in $(B \to K\pi)$ decays

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Abstract. We present a simple isospin invariant parametrization for $(B \to K\pi)$ decay amplitudes which consistently includes CP violation and (quasi-elastic) hadronic final state interactions. We find that the observed $(B \to K\pi)$ decays do not lead to a significant bound on the angle γ of the unitarity triangle. On the other hand, we claim that a sizeable CP violation asymmetry in $(B^\pm \to K\pi^\pm)$ rates is by no means excluded.

1 Introduction

The Standard Model (SM) encodes a very neat parametrization of quark mixing and CP violation through the Cabibbo-Kobayashi-Maskawa matrix (CKM matrix). At present, all existing data are consistent with this parametrization although precise experimental tests of the pattern of CP violation are still lacking. With B-factories forthcoming, the situation will soon improve and, at least in principle, detailed checks of the SM predictions for CP violation will become possible.

With this exciting perspective in mind, a huge amount of work has been devoted in recent years to possible ways of extracting information on CP violation from various B decays [1]. The difficulty is of course what to do with the "hadronic complications" which are unavoidable when physical parameters, precisely defined at the quark level, have to be related to measurable quantities in the hadronic world.

In this note, we reconsider the $(B \to K\pi)$ decay amplitudes and advocate the use of a hadronic basis, namely the *isospin* basis. This approach provides a simple bookkeeping procedure for (quasi-elastic) hadronic final state interaction effects. As a direct result we find that the observed $(B \to K\pi)$ decays do not lead to a significant bound on the γ angle of the unitarity CKM triangle without specific unwarranted assumptions on "hadronic effects". On the other hand, we emphasize that the CP asymmetry in $(B^\pm \to K\pi^\pm)$ can be quite large.

2 Isospin amplitudes

Isospin is a good symmetry of the hadronic world! In lowest order the SM weak Hamiltonian responsible for the $(B \to K\pi)$ decays contains both an isosinglet (H_W^0) and an isotriplet (H_W^1) part. The B mesons (B^+, B^0) are of course an isodoublet while the $(K\pi)$ system is a mixture of $I = 1/2$ and $I = 3/2$ eigenstates.

Let

$$a_1 \equiv \langle\langle B|H_W^0|(K\pi), I = 1/2\rangle\rangle \tag{1.a}$$
$$b_1 \equiv \langle\langle B|H_W^1|(K\pi), I = 1/2\rangle\rangle \tag{1.b}$$
$$b_3 \equiv \langle\langle B|H_W^1|(K\pi), I = 3/2\rangle\rangle \tag{1.c}$$

be the reduced matrix elements of the weak Hamiltonian. Let us now naively make the quasi-elastic (i.e. $SU(2)$-elastic) approximation for the $(K\pi)$ system: by Watson's theorem all final state interaction effects are then described by δ_1 and δ_3, the s-wave phase shifts in the $I = 1/2$ and $I = 3/2$ channels.

From all these old fashioned trivialities, one readily obtains

$$A(B^+ \to K^0\pi^+) = \sqrt{\frac{2}{3}} a_1 e^{i\delta_1} + \frac{\sqrt{2}}{3} b_1 e^{i\delta_1} - \frac{\sqrt{2}}{3} b_3 e^{i\delta_3} \tag{2.a}$$
$$A(B^0 \to K^+\pi^-) = \sqrt{\frac{2}{3}} a_1 e^{i\delta_1} - \frac{\sqrt{2}}{3} b_1 e^{i\delta_1} + \frac{\sqrt{2}}{3} b_3 e^{i\delta_3} \tag{2.b}$$

and similar expressions for the other channels.

We emphasize that in the quasi-elastic approximation the amplitudes a_1, b_1 and b_3 are relatively real.

3 Quark diagrams

The isospin invariant amplitudes a_1 and $b_{1,3}$ receive contributions from various SM quark diagrams. It is only at the level of these diagrams that the specific CP violation pattern of the SM can be correctly implemented. On the other hand all QCD effects are isospin invariant and can thus be ignored for our purposes.

For simplicity, let us only keep the contributions to (a_1, b_1, b_3) coming from the so-called [1] tree-level (T), color-suppressed (C) and QCD-penguin (P) quark diagrams.

Thus we write

$$a_1 = a_1^T e^{i\gamma} + a_1^C e^{i\gamma} + a_1^P + \cdots \tag{3}$$

and similar expressions for b_1 and b_3. In (3), γ is of course the CP-violating phase coming from the V_{ub}^* CKM matrix element while the dots stand for neglected contributions such as the annihilation amplitude.

It is now straightforward to derive the relations

$$a_1^T = -\sqrt{3} b_1^T \ , \quad b_3^T = -2 b_1^T \tag{4.a}$$
$$a_1^C = 0 \ , \quad b_3^C = b_1^C \tag{4.b}$$
$$b_1^P = 0 \ , \quad b_3^P = 0 \ . \tag{4.c}$$

We redefine

$$T \equiv -2\sqrt{2} b_1^T \tag{5.a}$$
$$C \equiv \sqrt{2} b_1^C \tag{5.b}$$
$$P \equiv \sqrt{\frac{2}{3}} a_1^P \tag{5.c}$$

and multiply (2) by an overall phase $e^{-i\delta_1}$ to obtain finally

$$\tilde{A}(B^+ \to K^0 \pi^+) = \frac{1}{3}(1 - e^{i\delta})(T + C)e^{i\gamma} + P \tag{6.a}$$
$$\tilde{A}(B^0 \to K^+ \pi^-) = \frac{1}{3}(1 - e^{i\delta})(2T - C)e^{i\gamma} + T e^{i\gamma} e^{i\delta} + P \tag{6.b}$$

where $\delta = \delta_3 - \delta_1$.

Equations (6) consistently include both CP violation as prescribed in the SM and quasi-elastic final state interactions as constrained by isospin invariance.

Note in particular that (6) are *not* equivalent to a commonly used [1] quark parametrization where T and P are given strong phases δ_T and δ_P, respectively. The latter parametrization is not compatible [2] with isospin invariance unless $\delta = \delta_3 - \delta_1 = \delta_T - \delta_P = 0$!

4 Comments and applications

The presence of a color-suppressed amplitude C in (6) confirms that the "quasi-elastic" rescatterings

$$B^+ \to \{K^+ \pi^0\} \to K^0 \pi^+ \tag{7.a}$$
$$B^0 \to \{K^0 \pi^0\} \to K^+ \pi^- \tag{7.b}$$

are correctly included in our formalism. So, we do not have to invoke penguin topology [3] with internal up-quark exchange: in the quasi-elastic approximation P is a real amplitude while $e^{i\delta}$ and $(1 - e^{i\delta})$ consistently approximate in an isospin invariant way final state interactions.

Color-allowed penguin amplitudes P_{EW} are second order weak effects. At this order, the weak Hamiltonian acquires, in general, an extra $I = 2$ piece, H_W^2, with reduced matrix element

$$c_3 \equiv \langle\langle B | H_W^2 | (K\pi), I = 3/2 \rangle\rangle \ . \tag{8}$$

However, the dominant electroweak diagrams (with a top intermediate state) have only $I = 0$ and $I = 1$ pieces and are thus easily included in (6) via the substitution

$$C e^{i\gamma} \mapsto C e^{i\gamma} + P_{EW} \ . \tag{9}$$

In the applications to follow, let us however neglect these potentially large contributions.

4.1 The Fleischer-Mannel (FM) bound

In the approximations made, we now define the (real) ratio

$$r = \frac{T}{P} \tag{10}$$

and consider

$$R \equiv \frac{\Gamma(B^0 \to K^+ \pi^-) + \Gamma(\bar{B}^0 \to K^- \pi^+)}{\Gamma(B^+ \to K^0 \pi^+) + \Gamma(B^- \to \bar{K}^0 \pi^-)} \tag{11}$$

recently measured by the CLEO Collaboration [4] to be $R = 0.65 \pm 0.40$.

From (6) one easily obtains the constraint

$$\sin^2\gamma \leq 1 - \frac{(1 - R)[5 - 2R + 2(2 + R)\cos\delta]}{[2 - R + (1 + R)\cos\delta]^2} \tag{12}$$

Obviously for $\delta = 0$, but only in this case, one recovers the FM bound [5], namely $\sin^2\gamma \leq R$. Clearly this latter bound is in general not valid and the constraint on γ given in (12) does depend on hadronic physics via the free parameter δ.

In other words, even if R turns out to be strictly less than unity, this does by no means exclude any value of the angle γ of the unitarity triangle, including $\gamma = \pi/2$!

4.2 CP asymmetry in $B^\pm \to K\pi^\pm$

Again a simple calculation gives

$$a \equiv \frac{\Gamma(B^+ \to K^0 \pi^+) - \Gamma(B^- \to \bar{K}^0 \pi^-)}{\Gamma(B^+ \to K^0 \pi^+) + \Gamma(B^- \to \bar{K}^0 \pi^-)} \simeq \frac{2}{3} r \sin\gamma \sin\delta \tag{13}$$

if QCD-penguin dominates this $B \to K\pi$ channel (i.e. $r < 1$). Clearly this asymmetry could be very sizeable contrary to recent claims [6] based on a quark parametrization with $\delta_{T,P}$.

Needless to say, our conclusions on the FM bound and on the $B^{\pm} \to K^0\pi^{\pm}$ asymmetry are reinforced if one takes into account color-allowed electrowek penguin contributions. The estimate [1]

$$\left|\frac{T}{P}\right| = \mathcal{O}(0.2) \;, \quad \left|\frac{P_{EW}}{T}\right| = \mathcal{O}(1) \tag{14}$$

based on factorization and $SU(3)$ relations suggest that T and P_{EW} amplitudes are equally important. In such a case, the bound on $\sin^2\gamma$ becomes almost totally useless while the $B^{\pm} \to K\pi^{\pm}$ CP asymmetry can be as large as 10%.

5 Conclusion

The quasi-elastic approximation made in this note has the main advantage of being simple. In this approximation quark diagram contributions are relatively real. Clearly a more sophisticated analysis of the final state interactions is called for. In particular, inelastic rescattering effects may be significant. The data will eventually settle this question.

It is straightforward to extend our analysis to more and more approximate flavor symmetries. In particular, a possible bonus is that rescattering effects of the type $\{K\pi\} \rightleftharpoons \{\bar{D}D_s\}$ can then be treated as quasi-elastic within $SU(4)_f$.

To conclude, let us stress once again the main point of this note: isospin is an excellent symmetry of the strong interactions and parametrizations of various mesonic decay amplitudes should at least be compatible with it!

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